A Communication on the Drag Reduction Correlation Proposed by Astarita, Greco, and Nicodemo

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No one of the several methods for correlating drag reduction data (*1, 2, 4, 7, 8*) has been demonstrated to be the best, and the limitations on some are quite severe. Rodriguez, Zakin and Patterson (RZP) have shown (*7*) that their correlation will not satisfactorily correlate data for systems of polymers dissolved in polar solvents. A later correlation (*1*) proposed by Astarita, Greco and Nicodemo (AGN) however used data for the drag reducing system of ET 597 dissolved in water, a polar solvent. But will the AGN correlation work for other systems of polymers dissolved in either polar or nonpolar solvents? It is the purpose of this communication to present the results of a study of the AGN correlation for the polymer polyethylene oxide dissolved in both polar and nonpolar solvents. Also, other important characteristics of the AGN correlation will be discussed.

The AGN correlation was tested by means of regression analysis to fit the f/f_{pv} versus log $\nu/\nu_{0.5}$ data to a polynomial of the form

$$\log(\nu/\nu_{0.5}) = A_0 + A_1(f/f_{pv}) + A_2(f/f_{pv})^2 + A_3(f/f_{pv})^3 + \dots \quad (1)$$

The goodness of the fit was characterized by the standard error of the estimate and the average absolute deviation.

For PEO WSR-301 and PEO-coagulant dissolved in water, only a second-degree polynomial was used because the value for the sum of the squares of residuals, the criterion used to make the fit, did not change significantly when higher degree fits were made. Data for f/f_{pv} ranged from values of 0.4 to 0.9. The standard error of the estimate for f/f_{pv} for the combined data of both polyethylene oxides was 0.0557 and the average absolute deviation was 0.0436. These data are shown as Curve 1 in Figure 1. The curve given by Astarita et al. for ET 597 dissolved in water coincided with the curve for the above two polyethylene oxides within the accuracy of the data.

The data presented by Liaw (*3*) were used to test the AGN correlation for various molecular weight polyethylene oxides and Hydrin 200 dissolved in nonpolar solvents. The systems investigated were PEO WSR-301 in benzene, PEO WSR-35 in benzene, PEO-coagulant in benzene, and Hydrin 200 in toluene. The data for all these systems were fitted with a single correlating curve of the form given by Equation (1) and shown in Figure 1 as Curve 2. This curve differed significantly in shape and position from the curve for systems using water as the solvent. For the polynomial which was fitted to all the above nonpolar solvent data the standard error of the estimate for f/f_{pv} was 0.0754 and the average absolute deviation was 0.0575. When just PEO WSR-301 and PEO-coagulant in benzene data were tested, the correlating curve, shown in Figure 1 as Curve 3, can be seen to deviate even further from the curve for the same polymers dissolved in water. The quantitative results for f/f_{pv} for these two polymers in benzene were a standard error of the estimate of 0.0793 and an average absolute deviation of 0.0588.

When Patterson et al. (*5*) discussed the similarities of the RZP and the AGN correlations, they questioned the effect of polymer concentration on $1/\nu_{0.5}$ for different polymer solutions. The reciprocal of $\nu_{0.5}$ is a relative measure of solution relaxation time where $\nu_{0.5}$ is the value if ν at f/f_{pv} equal to 0.5. In the low concentration range where the solutions exhibit Newtonian behavior, Astarita et al. obtained the experimental relation of $1/\nu_{0.5} \propto C^{2.5}$ for ET 597 in water. The two polyethylene oxides (PEO-coagulant and PEO WSR-301) dissolved in water that were studied in the present investigation produced the experimental relation of $1/\nu_{0.5} \propto C^{1.3}$ (*6*). From existing molecular theories Astarita et al. reported that the exponent on the concentration should be 2.0. When the nonpolar solvent data were tested, no simple relation like those obtained for polar solvents was found. In fact, the log-log plot of $1/\nu_{0.5}$ versus concentration for nonpolar solvents produced a curve rather than a straight line as theory predicts and as polar solvent data produced.

Thus polar and nonpolar solvent systems appear to behave in a similar manner as seen in Figure 1, but differ fundamentally with respect to the relation between $1/\nu_{0.5}$ and concentration. The maximum deviation of the AGN curve shown in Figure 1 for polar solvents compared with nonpolar solvents is about 20%. It is concluded that neither the AGN nor the RZP correlation adequately correlate both polar and nonpolar solvent systems.

One obvious shortcoming of the AGN correlation is that at least 50% drag reduction must be obtained to

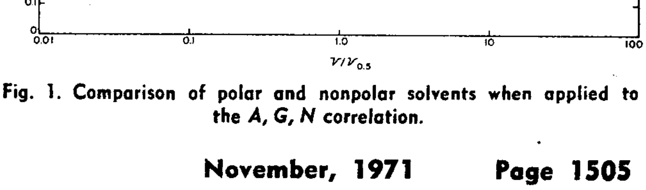

Fig. 1. Comparison of polar and nonpolar solvents when applied to the A, G, N correlation.

evaluate $\nu_{0.5}$. This may not be possible with some of the poorer drag reducing polymers. For example, guar gum in water was studied (*6*) at concentrations up to 100 ppm by mass and 50% drag reduction was not achieved. As a result, the system could not be correlated without data at higher concentrations where non-Newtonian behavior is approached.

The last point of consideration is the use of the parameter $\nu_{0.5}$ as opposed to a value of ν at some value of f/f_{pv} other than 0.5. The data for the polyethylene oxides (PEO-coagulant and PEO WSR-301) dissolved in water were used to examine this possibility. The results indicated that scatter about the correlating curve increased when the subscript on ν was increased incrementally from 0.5 to 0.8. Recall that the subscript on ν refers to the value of f/f_{pv} at which ν is evaluated. The value of ν at f/f_{pv} equal to 0.45 was tested ($\nu_{0.45}$) and showed some indication of decreasing this scatter. But the results were inconclusive because some data had to be extrapolated to obtain the value of $\nu_{0.45}$. The extrapolation was very susceptible to error, and values of $\nu_{0.45}$ varied greatly for different degrees of the polynomial used to fit the data for the plot of f/f_{pv} versus ν. Hence from a practical standpoint $\nu_{0.5}$ is probably the best parameter.

NOTATION

C = concentration, ppm by mass
D = tube inside diameter, ft.
f = Fanning friction factor
f_{pv} = Fanning friction factor for purely viscous fluids
N_{Re} = Reynolds number
U = bulk average velocity, ft./sec.
ν = characteristic frequency, $U/D \; N_{Re}^{0.75}$, sec.$^{-1}$
$\nu_{0.5}$ = value of ν for which $f/f_{pv} = 0.5$, sec.$^{-1}$

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Multiple Limit Cycles in a Continuous Stirred-Tank Reactor

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In their classical paper on the stability and control of continuous stirred-tank reactors (CSTR), Aris and Amundson (*1*) presented an example where there appeared to be multiple limit cycles. When they solved the nonlinear equations describing the CSTR on an analog computer for a particular set of parameters, they found a case where a stable limit cycle seemed to surround a stable equilibrium point. This kind of behavior can occur only when an unstable limit cycle lies between the stable singular point and the stable limit cycle, so that it acts as a source of trajectories for these two kinds of sinks. However, a more extensive analysis of this same problem by Luus and Lapidus (*8*) revealed that a stable limit cycle does not exist, that is, the real part of the root is sufficiently small that the trajectories appear to be closed, but actually they approach the stable equilibrium point very, very slowly.

It is possible to develop approximate analytical solutions for multiple limit cycles. One approach is simply to retain up to the fifth-order terms in the Taylor series expansions of the nonlinear functions presented in an earlier paper (*2*) so that one of the expressions for the secular terms becomes

$$C_4 A^4 + C_2 A^2 + C_0 A^2 = 0 \qquad (1)$$

where A represents the amplitude of the limit cycle, C_0, C_2, and C_4 are known constants which can be related to the CSTR design parameters, and C_4 depends on the fifth-order terms in the Taylor series expansions. Then, for certain values of the system parameters Equation (1) has two real roots, which hopefully correspond to the amplitudes of two limit cycles.

However, as has been shown before (*3*), different methods of approximating the process nonlinearities will lead to somewhat different solutions. Also, alternative generating solutions will correspond to different results. Hence, the region in parameter space where Equation (1) will predict multiple limit cycles will vary with the particular details of the solution technique. Regions of this type corresponding to three different approximate analytical solutions are shown in Figures 1 and 2 for two sets of system parameters. The first approximate solution is the one mentioned in the previous paragraph and is referred to as Gaitonde's method (*4*). The second is taken from the work of Gilles (*5*) and Hlaváček (*7*) et al. This solution is obtained by approximating the Arrhenius expression by the function e^{aT}, assuming a periodic solution for the reactor composition and temperature, using the series expansion for exp $(\cos \omega t)$ which contains Bessel functions as the coefficients in the series, and then undertaking a harmonic balance. The third approximate solution was developed by Heberling (*6*), who used the equations describing a curtate cycloid as a generating solution, rather than the more conventional trigonometric functions. All three solutions give a secular equation having the same form as Equation (1), and the

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